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The Symposium on Biodynamics Models and Their Applications took place in Dayton, Ohio, on 26-28 October 1970 under the sponsorship of the National Academy of Sciences - National Research Council, Committee on Hearing, Bioacoustics, and Biomechanics; the National Aeronautics and Space Administration; and the Aerospace Medical Research Laboratory, Aerospace Medical Division, United States Air Force. Most technical areas discussed included application of biodynamic models for the establishment of environmental exposure limits, models for interpretation of animal, dummy, and operational experiments, mechanical characterization of living tissue and isolated organs, models to describe man's response to impact, blast, and acoustic energy, and performance in biodynamic environments.

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PAPER NO. 21

THE EFFECT OF INITIAL CURVATURE ON THE DYNAMIC RESPONSE OF THE SPINE TO AXIAL ACCELERATION

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ABSTRACT

A majority of the studies on the dynamic response of the human torso have considered uni-axial models wherein the initial curvature of the spine is ignored. A detailed discrete parameter vertebral response model incorporating the variable geometry of the spine and subjected to pilot ejection simulated impact conditions has been recently investigated by Orne and Liu. In this work, a simple continuum representation of the spine is formulated and the resulting boundary value problem is solved for the axial and lateral (bending) dynamic response. The assumed model is a constant cross-section, sinusoidally curved, elastic beam with an end mass subjected to an axial acceleration at the other end. The effects of transverse shear and rotational inertia are ignored in the model. The equation governing axial displacement is a non-homogeneous wave equation subjected to non-homogeneous boundary conditions. The governing approximate equation for the lateral deflection is a non-linear second order differential equation with variable coefficients. Short time solutions for these equations are obtained to demonstrate the effect of initial curvature on the spinal dynamic response. Numerical results indicate that the dynamic bending stress is significant in comparison to the axial dynamic stress.

LIST OF SYMBOLS

Dimensional Quantity	Physical interpretation
A	effective spinal cross-sectional area
$c = \sqrt{E/\mu}$	compressional wave speed
E	instantaneous Young's modulus
$I = Ar^2$	principal moment of inertia
M	concentrated head and upper torso mass
r	effective radius of gyration about spinal bending axis
μ	lumped effective torso and spine mass density

Non-dimensional Quantity	To convert to dimensional form $(\bar{\ })$ multiply by	Physical Interpretation
a	c^2/r	forcing acceleration
L	r	effective spinal length
P	AE	axial force
t	r/c	time
u	r	axial column displacement
x	r	axial co-ordinate
$y(x,t)$	r	total column bending displacement
$Y(t)$	r	time function
$y_0(x)$	r	initial column bending displacement
Y_0	r	maximum column eccentricity
$\lambda = M/\mu Ar = \bar{\lambda}\,\bar{L}$		mass parameter

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INTRODUCTION

Crew member protection from hostile aerospace environments is a biomedical engineering problem of grave concern. Rheological and structural response models of human body system components have received particular attention towards defining limiting injury thresholds associated with the governing mechanism(s) of injury. Of special interest is the modelling of the vertebral column response to transient headward accelerations along the spinal axis (+ G_z ejection mode). Reported data on vertebral fractures resulting from pilot ejection reveals that a majority of these fractures occur between T8 and L1. Several discrete parameter and continuum models of the human torso ranging in complexity and scope have established the desirability of analytical representation of the response variable defining injury. A review of pertinent investigations can be found in studies by von Gierke [1], Roberts, et al [2], and Orne and Liu [3]. Selected contributions are indicated below.

Uniaxial spring mass characterizations of the human torso under impact have been examined by Latham [4], Payne [5], Stech [6], and others [7]. A more refined model described by an eight degree of freedom damped spring mass system has been studied by Toth [8]. Recently, Orne and Liu [3] have investigated a detailed multi-mass representation of the torso incorporating the effects of spinal disk axial, bending, and shear deformations in addition to the variable vertebral geometry. The discrete parameter models involve the (simultaneous) solution of ordinary differential equation(s) formulated from the conditions of dynamic equilibrium. Research on continuum descriptions of the torso includes one dimensional wave propagation models considered by Hess and Lombard [9], Liu and Murray [10], Liu [11], Terry and Roberts [12], and Murray and Tayler [13]. These uniaxial continuum models vary in their degree of sophistication depending on the boundary conditions (head mass), and constitutive relations (linear/non-linear, elastic/visco-elastic). It is noteworthy that experimental results and analytical solutions comparing the rectangular pulse response of an elastic rod-mass system with that of an equivalent spring mass approximation have been obtained by Seigel and Waser [14]. Their work indicates that the rod-mass system experiences "significantly larger" forces for short pulse duration and/or end mass magnitude.

In this paper, we consider a simplified continuum dynamic model representation of the curved spine with the torso mass uniformly distributed along its length. The idealized model is a constant cross-section, sinusoidally curved, elastic column with end mass subjected to a uniform acceleration at the other end. The influence of transverse shear and rotational inertia is ignored in the model. In addition, the effects of moments arising from the head-torso mass eccentricity and the external support-restraint system interaction are not included

in the analysis. The bending stress resulting from these external force and moment intensities can be superposed on the selected basic model under study. Experimental work by Vulcan, King and Nakamura [15] indicates the relative importance of support-restraint systems and head-torso rotation on bending stresses in the vertebral beam-column. The motivation for the assumed model stems from the results of small animal $+G_z$ impact experiments conducted with flat back and contoured support-restraint systems [16,17]. The large incidence of vertebral fractures and paralysis for the flat back system evidently supports the consideration of initial spinal curvature.

As a problem in theoretical mechanics, the non-linear dynamic response of a simply supported column with sinusoidal initial curvature and a constant velocity forcing function at one end has been studied by Hoff [18], Sevin [19], and Dym and Rasmussen [20]. A comprehensive study of the curved dynamic beam response under constant velocity end loading with combinations of simply supported and clamped boundary conditions has been conducted by Archer and Das [21]. They demonstrate an improved numerical stability of their finite difference solution when the effects of beam transverse shear and rotational inertia are included. Here, the equations and associated boundary conditions governing the axial and bending spinal column motion are formulated and uncoupled. The non-homogeneous wave equation for the axial motion is solved and an approximate equation for the bending response time variable is obtained by using the Ritz-Galerkin procedure. Short time solutions for the spinal response are obtained by the Runge-Kutta method to demonstrate the importance of initial curvature in considering the spinal column response.

FORMULATION OF THE BOUNDARY VALUE PROBLEM

The equations governing motion of the basic spinal model (Fig. 1) can be derived by use of Hamilton's principle. The Lagrangian, using this variational energy formulation, considers the strain and kinetic energies of the column and the work done by the axial force. The non-dimensional equations, in terms of the coupled generalized co-ordinates $y(x,t)$ and $u(x,t)$ with respect to an inertially defined co-ordinate system are:

(i) An equation governing the bending motion of the column

$$y'''' + (P'y' + Py'') + \ddot{y} = y_0'''' \tag{1}$$

(ii) An equation governing compressive motion of the column

$$-P' = \ddot{u} \tag{2}$$

The non-dimensional axial force $P(x,t)$ in equations (1) and (2) is defined by

$$P = -\left\{u' + \frac{1}{2}\left[(y')^2 - (y_0')^2\right]\right\} \tag{3}$$

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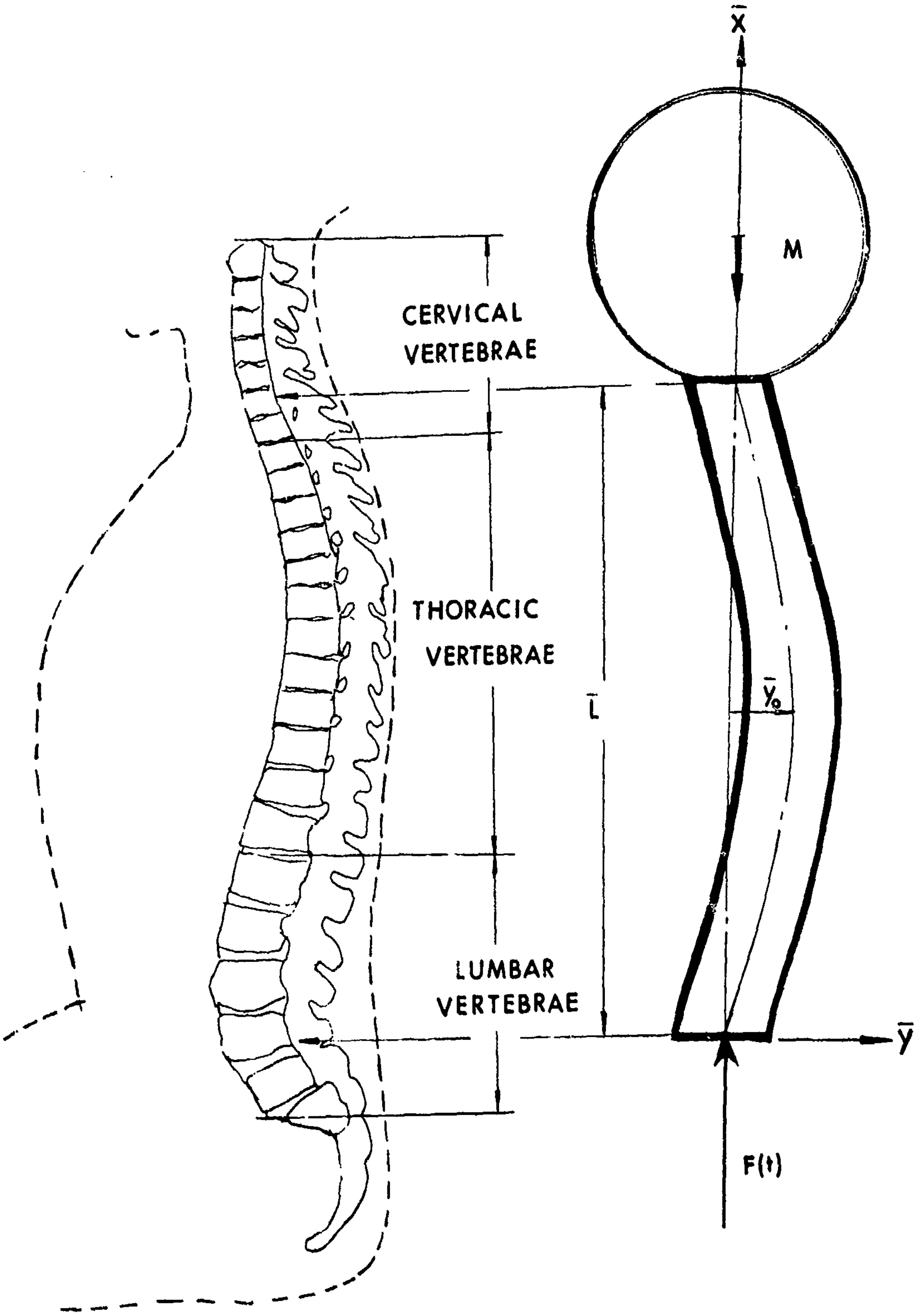

FIG. 1 Vertebral Response Model

In the preceding equations primes and dots denote differentiation with respect to the non-dimensionalized space and time variables respectivly. The influence of column transverse shear and column, torso-head rotational inertia is not considered in these equations. A more general representation of equations (1), (2), and (3) is currently under study at Technology Incorporated using the finite difference method [22].

The boundary and initial conditions for the assumed model are

$$u(0,t) = \frac{1}{2}at^2 \quad , \qquad P(L,t) = \lambda\ddot{u}(L,t) \tag{4a}$$

$$y(0,t) = y(L,t) = y''(0,t) = y''(L,t) = 0 \tag{4b}$$

$$u(x,0) = \dot{u}(x,0) = \dot{y}(x,0) = 0 \quad , \qquad \text{and } y(x,0) = y_0(x) \tag{4c}$$

AXIAL DISPLACEMENT RESPONSE SOLUTION

Equations (2) and (3) can be re-written in the form

$$\ddot{u} - u'' = f'(x,t) \tag{5}$$

with $f(x,t) = \frac{1}{2}\left[(y')^2 - (y_0')^2\right]$.

The solution to equation (5) can be obtained by considering (i) a homogeneous wave equation subjected to non-homogeneous boundary conditions and (ii) a non-homogeneous wave equation with homogeneous boundary conditions. We therefore write

$$u = u_H + u_P \tag{6}$$

where u_H satisfies the equation

$$\ddot{u}_H - u_H'' = 0 \tag{7}$$

with the boundary and initial conditions

$$u_H(o,t) = \frac{1}{2}at^2 \quad , \qquad u_H'(L,t) = -\lambda\ddot{u}_H(L,t) - f(L,t), u_H(x,0) = \dot{u}_H(x,0) = 0$$

and u_P satisfies the equation

$$\ddot{u}_P - u_P'' = f'(x,t) \tag{8}$$

with homogeneous boundary and initial conditions

$$u_P(0,t) = u_P'(L,t) = u_P(x,0) = \dot{u}_P(x,0) = 0 \quad .$$

We obtain the solution to equation (7) in a manner similar to that of Liu and Murray [10] with the modification expressed by the term $f(L,t)$ in the boundary conditions. The resulting solution after use of the Laplace transform method is

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$$u_H(x,t) = \frac{a}{2}\Bigg\{(t-x)^2\, H(t-x) + \sum_{n=1}^{\infty}\Big[(t-2nL-x)^2\, H(t-2nL-x)$$

$$-(t-2nL+x)^2\quad H(t-2nL+x)\Big] + \sum_{n=1}^{\infty}\sum_{m=1}^{n}(-1)^m \frac{2^m\lambda^{-n}}{(m-1)!}\,{}^nC_m\,\cdot$$

$$\cdot\int_0^t \Big[(\tau-2nL-x)^2\, H(\tau-2nL-x) - (\tau-2nL+x)^2\, H(\tau-2nL+x)\Big]\cdot$$

$$\cdot\,(t-\tau)^{m-1}\, e^{-(t-\tau)/\lambda}\, d\tau\Bigg\}$$

$$-\sum_{n=1}^{\infty}\int_0^t (-1)^n\Big[f(L,\tau+x-(2n+1)L) - f(L,\tau-x-(2n+1)\,L\Big]\cdot$$

$$\cdot\left\{\mathcal{L}^{-1}\left[\frac{\varepsilon^n(s)}{s}\right] - \lambda\mathcal{L}^{-1}\left[\frac{\varepsilon^n(s)}{1+\lambda s}\right]\right\}d\tau \tag{9}$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform and

$$\varepsilon(s) = (1-\lambda s)/(1+\lambda s)\quad .$$

The solution to the non-homogeneous equation (8) can be obtained in the form

$$u_P = \sum_{n=1,3,5}^{\infty}\frac{4}{n\pi}\sin\frac{n\pi x}{2L}\int_0^t\Bigg\{f(L,t)\sin\frac{n\pi}{2}$$

$$-\frac{n\pi}{2L}\int_0^L f(x,\tau)\cos\frac{n\pi x}{2L}\,dx\Bigg\}\sin\frac{n\pi}{2L}(t-\tau)\quad d\tau \tag{10}$$

SOLUTION FOR THE BENDING RESPONSE

Equation (1) governing the column lateral response $y(x,t)$ is simplified by selecting the initial column deviation $y_0(x)$ from the vertical axis to be sinusoidal. Additionally, in view of boundary conditions (4b) we assume that the column responds in the first spatial mode. We therefore take

$$y(x,0) = y_0(x) = Y_0\sin(\pi x/L) \tag{11}$$

$$y(x,t) = Y(t)\sin(\pi x/L) \tag{12}$$

Substituting equations (3), (6), (9), (10), (11) and (12) in equation (1) and using the Ritz-Galerkin averaging method, we obtain after considerable simplification a non-linear, variable coefficient, second order differential equation governing $Y(t)$. It is

$$\ddot{Y}(t) + \left(\frac{\pi}{L}\right)^4 \left[(Y - Y_0) + \frac{3}{8}(Y^2 - Y_0^2)\, Y + \left(\frac{L}{\pi}\right)^4 \cdot \frac{2}{L} A(t)\; Y - \frac{64}{\pi L}\, Y \sum_{n=1,3}^{\infty} \frac{n^2-4}{n(n^2-16)^2} G_n(t) \right] = 0 \tag{13}$$

where

$$A(t) = -\left(\frac{\pi}{L}\right)^2 \int_0^L P_H \sin^2 \frac{\pi x}{L}\, dx + \frac{\pi}{L} \int_0^L P_H' \sin\frac{\pi x}{L} \cos\frac{\pi x}{L}\, dx$$

$$G_n(t) = \int_0^t (Y^2 - Y_0^2) \sin \frac{n\pi}{2L} (t - \tau)\, d\tau$$

and P_H is the axial force derived from the homogeneous wave equation.

Asymptotic solutions to second order non-linear differential equations with variable coefficients of the type designated by equation (13) have been investigated by Kuzmak (23). However, these solutions are valid for slowly varying time coefficients. A power series solution to (13), with physical constants represented by spinal constitutive and geometric properties and a forcing acceleration ($\ddot{u}$) of 20 G, exhibited numerical instability following a time duration of 15 milliseconds. An improved technique of solution using the Runge-Kutta method in conjunction with an iterative procedure was finally used to yield the transient model response.

ASSUMED SPINAL CONSTITUTIVE AND GEOMETRIC PROPERTIES

An extensive literature review revealed that available data is inadequate for characterizing the spinal response in the short time domain. Data surveyed included results on spinal compressive wave propagation experiments and analysis [6,7,22], natural axial frequency data on spinal response [1,7,24], and compression and bending tests on human vertebrae and disks [22,25]. The wide range of reported results is evidenced by comparing (i) a calculated compressive wave velocity of 100 ft/sec obtained by Hess and Lombard [9] versus an experimentally determined spinal cadaveric velocity of 191 ft/sec [22] and (ii) a first resonance axial frequency of around 10 Hz for the spine-upper torso mass [1] as compared to 44 Hz indicated in another study [7]. The static compressive properties of vertebrae and disks are well documented in the literature. Experiments conducted at Technology Incorporated [22] on the compressive response of human vertebrae and disks, using linear visco-elastic theory, indicate that "the average initial elastic modulus" for sets of one bone

plus one disk is 7428 psi. The corresponding reported values for vertebrae and disks are 10,029 psi and 2552 psi respectively [22]. Preliminary results from static moment-curvature tests on human cadaveric spines reveal that its flexural rigidity EI ranges from 6×10^3 to 10^4 lb-in^2 [25].

Based on the data and literature reported in the preceding paragraph, a compromised set of constitutive and geometric constants was selected. The values chosen for the assumed model are:

Effective spinal length (L-4 to Cervical Vertebrae) $\bar{L} = 18$ in

Effective cross-section of area $A = 1.8$ in^2

Effective radius of gyration about bending axis $r = 0.527$ in

Spinal column eccentricity $\bar{Y}_0 = 2$ in

Head and upper torso concentrated mass $M = 0.055$ lb sec^2/in

Non-dimensional mass parameter $\bar{\lambda} = M/\mu A \bar{L} = 0.33$

<u>Instantaneous</u> elastic modulus $E = 10{,}000$ psi

The undamped compressive wave velocity and axial spinal frequency with the above data are 120 ft/sec and 13.5 Hz respectively. The spinal cross-sectional area and radius of inertia take into account the added contribution of the supporting vertebral structure. In addition, based on the "hardening" strain rate characteristics of most biological materials, the assumed instantaneous model elastic modulus is chosen to be larger than reported static values.

DISCUSSION AND NUMERICAL RESULTS

Before proceeding to illustrate the results of the numerical computations, a discussion of the assumed model and its inherent limitations will be presented. The selection of a simple half sine wave for describing the initial spinal configuration deserves special mention. Based on geometric data, the sine wave adequately defines the spinal curvature from the cervical vertebrae to the upper lumbar region. The results of Orne and Liu [3] demonstrate a vanishing bending moment for durations up to 90 milliseconds in the vicinity of the L-3 region, thereby justifying the assumed deflection form. In addition, their results indicate that the axial force remains relatively constant in the lumbar region for a specified instant. The continuum model, being an initial attempt towards demonstrating the effect of spinal curvature, neither includes the bending moment contribution of the head-torso mass eccentricities nor the influence of the support-restraint system interaction. These effects can be introduced by refining the model to a beam-column subjected to external distributed dynamic moments and lateral forces (incorporated as a rotational inertia term $\mu J_0 y_{xxtt}$ and external forcing term $q(x,t)$ in the beam equation). The effective mass terms participating in the bending and axial modes would also require modification in this model.

Numerical work was performed on an IBM 360, Model 75 computer. The Runge-Kutta routine was combined with an iterative procedure to compute

the lateral deflection. Figure 2 illustrates the mid-span lateral deflection time history computed from equation (15) for acceleration loadings of a = 10 G and 20 G. The corresponding total compressive axial displacement is also indicated in this figure. This axial displacement is a superposition of solutions obtained from (i) a homogeneous wave equation (7) with non-homogeneous boundary and initial conditions and (ii) a wave equation (8) with a forcing acceleration term and homogeneous boundary and initial conditions. Since the support-restraint reactions are ignored in the analysis, the results are assumed to be relevant up to a time duration of 40 milliseconds. Figure 3 compares the axial force computed for the uniaxial and axial-bending response models at the load and head ends. The significantly reduced dynamic response factor at the forcing end results from the curvature terms. The head end axial force is also reduced and tends to be tensile for larger elapsed times. The maximum compressive fiber stress time history at the anterior mid-span obtained from the relation $\sigma = -(P/A + Md/I)$ is shown in Fig. 4. The assumed initial deflection form presupposes a maximum bending contribution at the mid-span (T-8 or T-9) which increases with time in the interval considered. This is in contrast with results which indicate a reversal of sign in the bending moment around this neighborhood [3]. The bending moment stress contribution due to the initial curvature is about 30 percent at t = 40 milliseconds. This effect would be further enhanced if the moments arising from the rotation of the head and movement of the torso were considered. The instantaneous elastic response solutions in the figures represent upper bounds for the visco-elastic vertebral model. However, the early time response for the elastic and damped models is almost identical.

A comprehensive discussion of the mechanisms associated with vertebral injury has been presented by Kazarian, et al [26]. Among these the anterior lip fracture, the compression fracture, and the hyperextension fracture are of particular interest. Coupled with these findings is the reported constitutive experimental data [27,28,29]. In the tests conducted by Crocker and Higgins [29], the intervertebral disks exhibited "hardening" stiffness properties with increasing strain rate. However, their maximum compressive velocity rate of 4mm/sec is well below the corresponding rate encountered in vertebral ejection. Based on the general trend of available experimental results and the model analysis represented by Figs. 2,3, and 4 the mechanisms of injury indicated above lend themselves to analytical definition. For example, data on anterior lip, compression and hyperextension fractures can be correlated with computed dynamic values of compressive stress, tensile stress (due to excessive bending moment) and/or bending and axial displacements. The spinous process fracture with displacement of the pedicle may be interpreted by incorporating the effects of transverse shear in the governing equation.

CONCLUSIONS AND RECOMMENDATIONS

The geometrically non-linear, continuum model analyzed here answers some basic questions pertinent to the interpretation and prediction of vertebral column injury resulting from dynamic axial loads. Specifically, the initial spinal curvature introduces a coupled axial-lateral response.

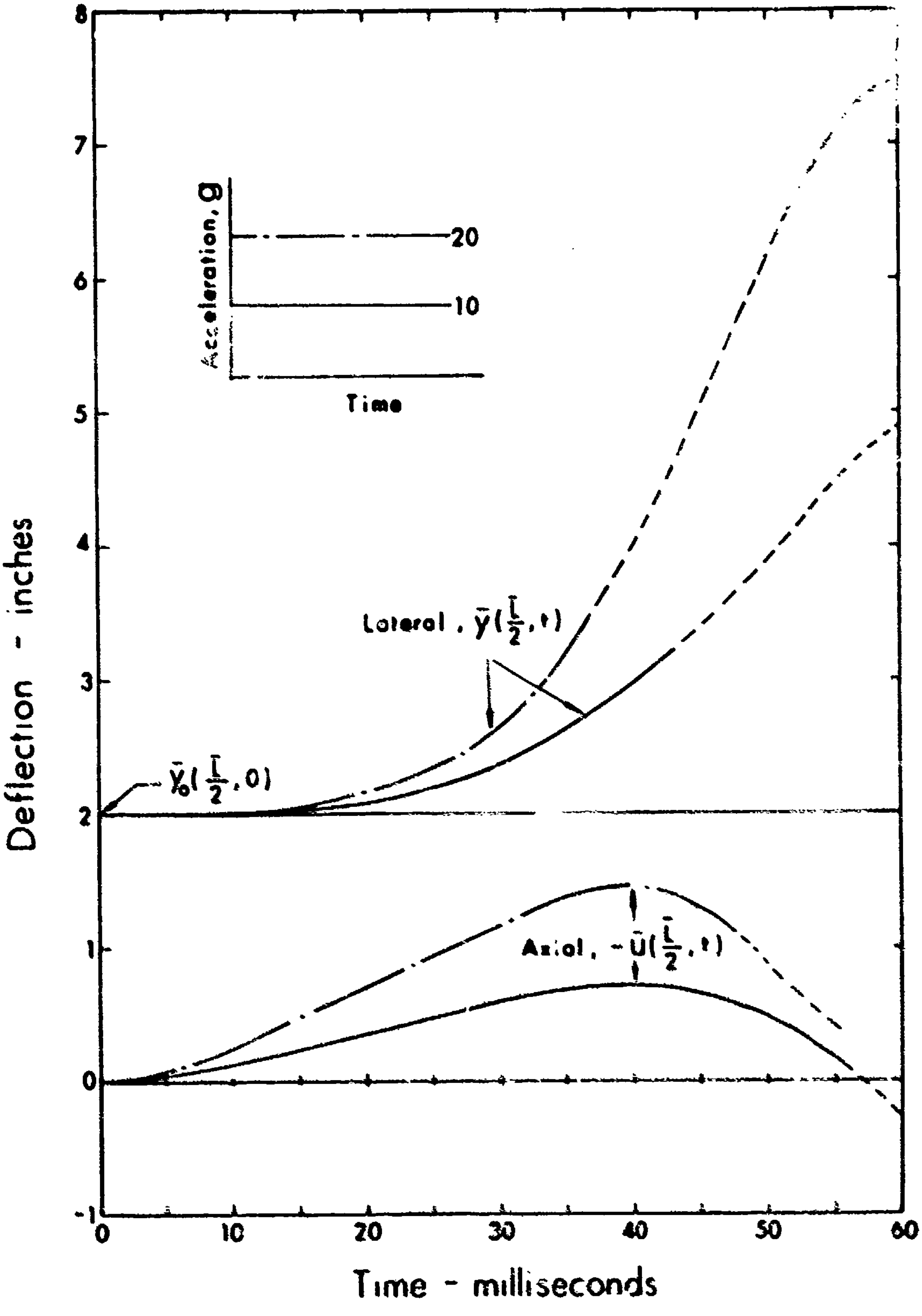

FIG. 2 Mid-span Deflections

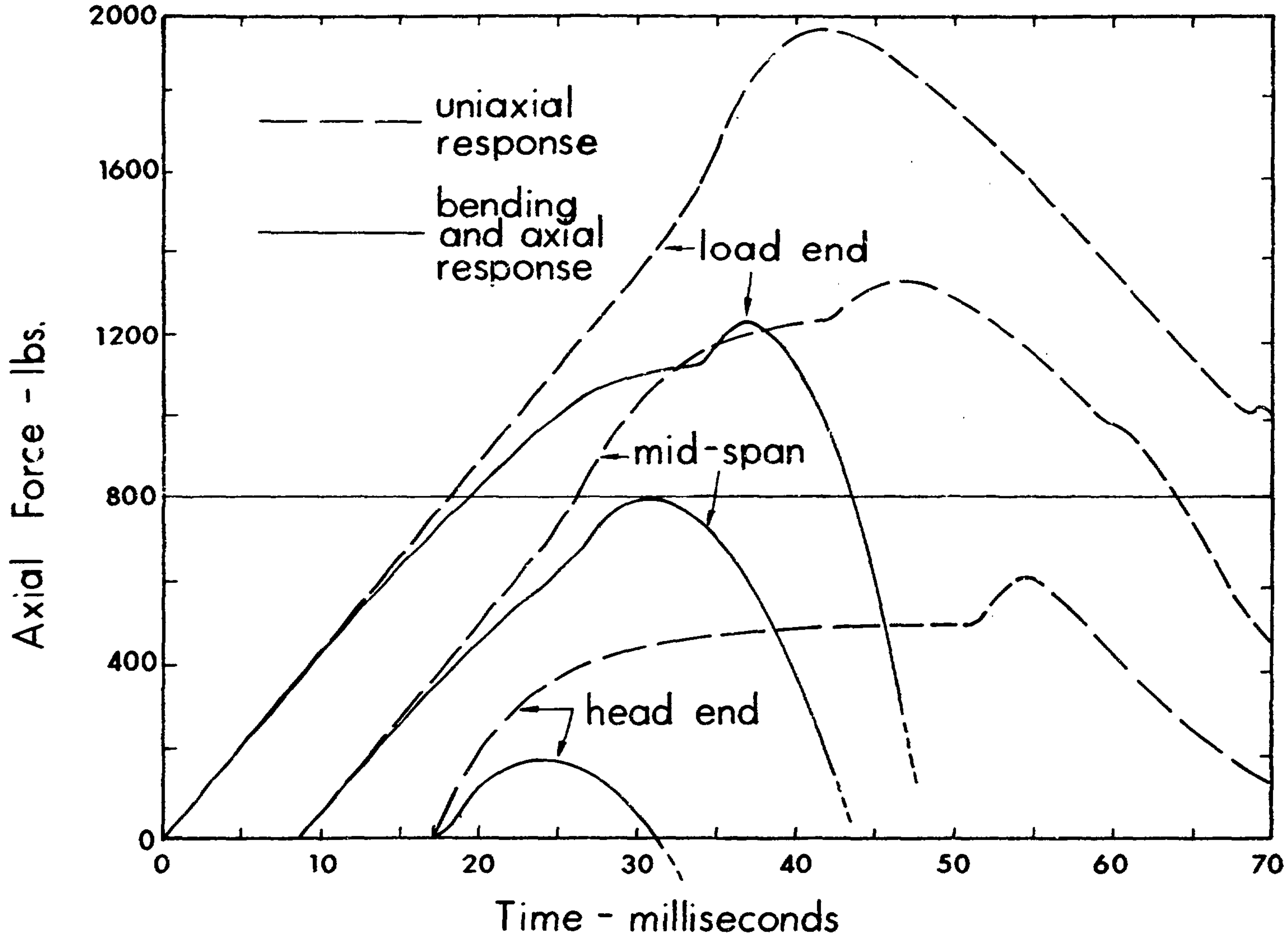

FIG. 3 Axial Force vs. Time, 10g

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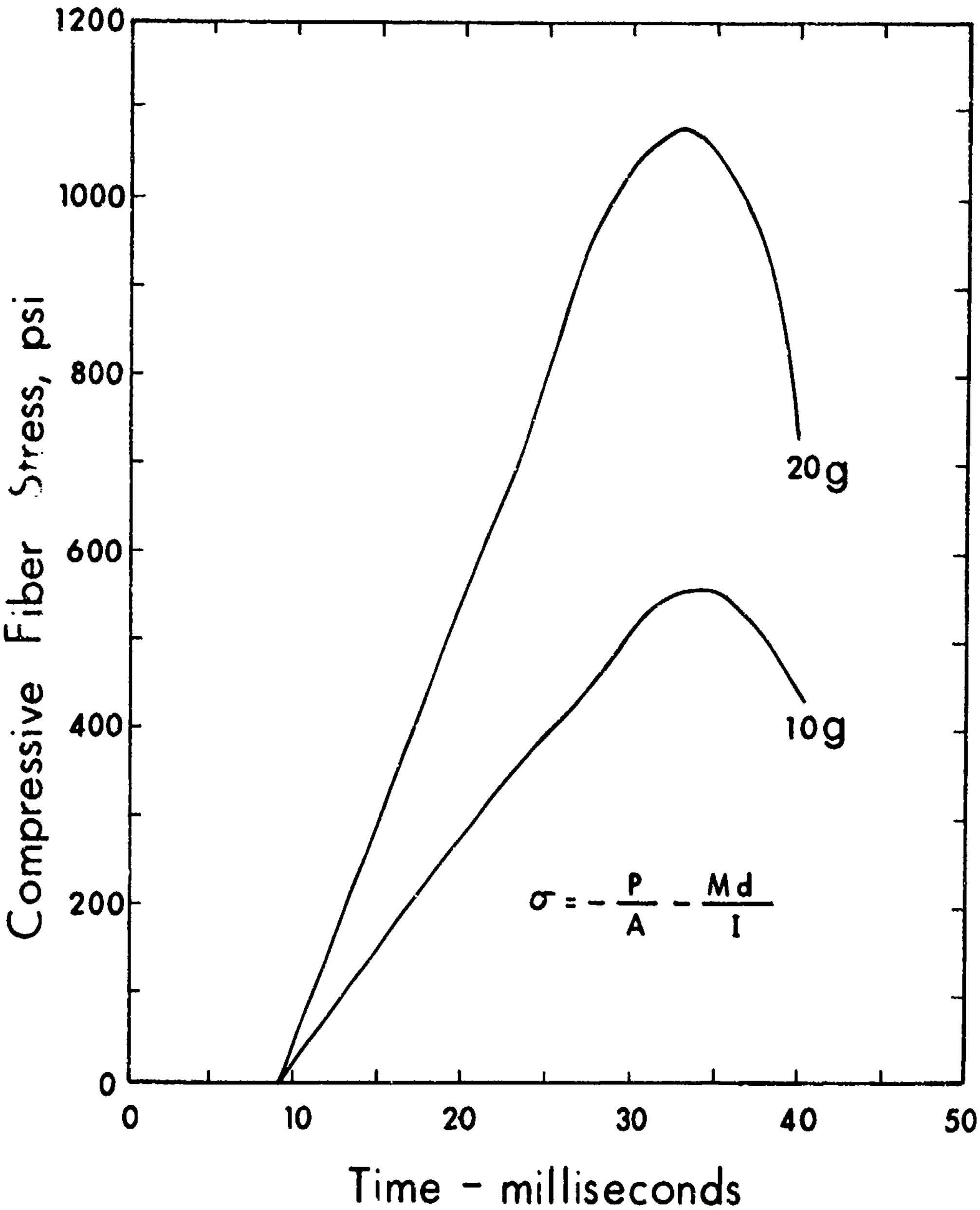

FIG. 4 Mid-span Maximum Fiber Stress

The axial force and stress distribution are significantly influenced by the bending motion response as a result of the mechanical energy distribution between the axial and bending modes.

It is recommended that spinal disk constitutive equations valid in the impact range be determined from high strain rate compressive tests. Finally, it is suggested that more complex continuum models be formulated and the resulting boundary value problem numerically studied to examine the detailed spinal stress response, the torso surface wave response and the associated mechanical energies producing injury.

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SESSION IV

MODELS TO DESCRIBE MAN'S RESPONSE TO IMPACT, BLAST, AND ACOUSTIC ENERGY

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